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STUDY OF THE DISTRIBUTION OF INTERPLANETARY AND STELLAR DUST MASSES

French title on bottom of next page

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STUDY OF THE DISTRIBUTION OF INTERPLANETARY AND STELLAR DUST MASSES *

C. R. Académie des Sciences
Astrophysique
T. 258, pp. 82-85,
Paris, 6 Jan. 1964.

Note by M. Maurice Surdin

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SUMMARY

Starting from two plausible hypotheses, a law of interplanetary dust distribution as a function of mass is obtained. This law accounts very well for the experimental results. In the second part of this Note a state of equilibrium of an aggregate of bodies in interaction with mass exchange is considered. A law of distribution, corresponding to that of stars is derived.

* * *

1.- The study of interplanetary dust offers multiple aspects. The collection on the ground of spherules assumed to be of cosmic origin, and the study of their distribution as a function of their mass allows, among other things, to estimate the accretion rate of the terrestrial mass [1, 2, 3]. The study of interplanetary dust with the aid of satellites allowed to estimate the danger this dust constitutes for spacecrafts and to formulate certain hypotheses on the nature of the lunar ground [4, 5, 6, 7, 8]. Certain studies on the nature of the zodiacal light are based upon solar light diffusion by that dust [9, 10].

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* [Etude de la distribution des masses de poussieres interplanétaires et de celles des étoiles.]

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The laws of distribution of meteors, meteorites and asteroids as a function of their masses are studied with the view of increasing our knowledge of the solar system [11, 12, 13, 14].

It is interesting to note that the distribution laws found empirically, for the different types of dust mentioned above [15] are all of the same form [6], say

$$df = \frac{C}{m^a} dm, \qquad (1)$$

where df is the relative number of dust particles of mass m, with an approximation to dm, C is a constant and a is a number near 2.

Since $a > 1$, the quantity of matter, mdf per mass interval dm, increases as the mass of dust particles decreases.

If we admit that the small dust particles are obtained from the larger ones by ablation, there would be accumulation of matter in the domain of small masses and empoverishment in that of great masses. The Poynting-Robertson effect, i.e. the evacuation of dust particles of tiny dimensions because of radiation pressure towards the border of the solar system rules out the accumulation of matter in the domain of feeble masses. If one considers that the actual state is stationary, it must be admitted that there exist particles of a very heavy mass; this role could be attributed for example to debris of the missing planet.

It is admitted in the following that a dust particle is obtained by ablation or fragmentation of a more massive dust particle. To each ablation event a particle of mass m loses as an average a mass δm, which we assume to be proportional to m, say

$$\delta m = -\mu m, \qquad (2)$$

μ being a very small number, independent from m; it could depend on the dust velocity and its cohesion energy per unit of mass. After n abalation events, the dust mass becomes

$$m' = m_0 e^{-\mu n}. \qquad (3)$$

On the other hand, at each event the number of dust particles N increases by δN as an average; we admit that δN is proportional to N, say

$$\delta N = \nu N, \tag{4}$$

where ν is a very small number; it is the ratio of the mean duration between to successive events to the average lifetime of the dust particle.

After n events the number of dust particles becomes

$$N = N_0 e^{\nu n}. \tag{5}$$

From the equations (3) and (5) we derive

$$df = \frac{\nu}{\mu} \frac{(m_1)^{\frac{\mu}{\mu}}}{m^{1+\frac{\nu}{\mu}}} dm, \tag{6}$$

where m_1 is the minimum distribution mass.

The expression (6) has the same form as the empirical expression (1), their identification amounting to postulate that $\nu/\mu \cong 1$.

2.- In this part of the Note we shall study the distribution at the equilibrium of an aggregate bodies in interaction with mass exchange, but whose total masss remains constant; the particular interaction mechanism is not specified.

We search for the most probable mass distribution between the various bodies of the aggregate, the following demonstration being inspired by the one that allows the obtention of the Maxwell-Boltzmann distribution law [16].

The in-phase extension element corresponding to the system being studied is proportional to Δm. A certain state of the aggregate is considered, and let n_i be the number of bodies which, in such a state, has masses ranging from m_i to $m_i + \Delta m_i$. The total mass of the aggregate is $M = \sum n_i m_i$. Let g_i be the number of states, a priori possible, of the sub-aggregate corresponding to the interval m_i, $m_i + \Delta m_i$; this number is proportional to the in-phase extension element corresponding yo that mass interval, $g_i = \text{const.} \cdot \Delta m_i$.

The probability of the state considered is proportional to the number of ways to distribute N objects between various mass intervals, say to

$$N!\prod_i \frac{1}{n_i!},$$

it is also proportional to the number of ways equal to $g_i^{n_i}$ to distribute n_i bodies of each interval between the g_i corresponding states; we have

$$P = \text{Cte.}\, N!\prod_i \frac{g_i^{n_i}}{n_i!} \qquad (7).$$

We assume that the m_i are sufficiently small to make possible to substitute integrals for the sums, but sufficiently great for the n_i to be great and for being able to apply the Stirling formula. We have

$$\log P = \sum_i n_i(\log g_i - \log n_i + 1) + \text{Cte.} \qquad (8).$$

The determine the most probable state of the aggregate, we search for the maximum of log P, taking into account the fact that the total mass and the total number of bodies of the aggregate are constant :

$$\begin{cases} 0 = \delta \log P = \sum_i \delta n_i (\log g_i - \log n_i), \\ 0 = \delta M = \sum_i \delta n_i m_i, \\ 0 = \delta N = \sum_i \delta n_i. \end{cases} \qquad (9)$$

Utilizing the method of Lagrange multipliers, we have

$$0 = \delta \log P - \alpha \delta N - \beta \delta M = \sum_i \delta n_i (\log g_i - \log n_i - \alpha - \beta m_i),$$

whence

$$n_i = g_i e^{-\alpha - \beta m_i} = A e^{-\beta m_i} dm_i \qquad (10).$$

and

$$df = \frac{n_i}{N} = C e^{-\beta m} dm. \qquad (11)$$

The constants C and β are determined with the aid of the following relations

$$\int_0^\infty df = 1 \quad \text{et} \quad m_0 = \int_0^\infty m\, df,$$

where $m_o = M/N$ is the average mass.

We finally have

$$df = \frac{1}{m_0} e^{-\frac{m}{m_0}} dm. \qquad (12)$$

Let us compare the distribution (12) with that of stars derived from the experience. We obtain the distribution of stars as a function of their absolute brightness in magnitude, at least for the stars near the Sun, for which experimental data are available [17]. To that effect we utilize an empirical relation linking the absolute brightness in energy with the star's mass. If we plot the thus found experimental points in semi-logarithmic coordinates together with a straight line derived from the equation (12), we may note that the agreement is good with $m_o = 0.5\ m_\odot$.

We must note the uncertainty that exists over the exprimental results giving the number of stars as a function of their brightness and the relation which links star's brightness to its mass. Despite this, it would be interesting to compare the distribution law (12) to the experimental distribution of stars of the Galaxy, and, eventually, with the stars of the Universe. Such a comparison could provide indications on the state of equilibrium of the Universe, at least with an approximation tk the radiation emission, and on the constance of the number of stellar objects.

***** THE END *****

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